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Biaxial Anisotropy in Geoelectric Prospecting

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Abstract—In applied studies on electromagnetic well-logging and electric prospecting, a medium is supposed to be either isotropic or with conventional anisotropy of conductivity. However, in the meantime, there is a clear hint that a medium may exhibit even biaxial anisotropy; i.e., the resistances along all three directions *X*, and *Y*, *Z* are different. Based on the analytical solution by the method of separation of variables, the paper considers an algorithm for the calculation of the electromagnetic field in a layered medium with biaxial conductivity anisotropy involving an arbitrary direction of horizontal conductivities in each layer. In this case, the theoretical solution and the algorithm display substantial peculiarities, and the numerical implementation involves many complexities. These problems were examined and solved mainly due to the constant comparison with the calculations carried by the finite element method and their analysis. Programs for the calculation of electromagnetic fields in the frequency and time domains were developed, and results of geoelectric interest were obtained. For example, the excitement of a horizontally-layered section by a magnetic dipole produces a vertical electric component of the field.

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1. INTRODUCTION

Until now, only conventional anisotropy has been allowed for in mathematical modeling in problems of geoelectric researches both in the design stage and during processing of field data. The horizontal resistance is assumed to be the same in all the directions. However, there are many indications that a medium may exhibit even biaxial anisotropy; i.e., the resistances along all three directions *X*, *Y*, and *Z* are different. Properly speaking, when we come to think about it, the dependence of the horizontal resistance on the direction is just natural. The sedimentation (development of formations) is always accompanied by directional prevailing factors like atmospheric motions and water flow. In addition to this, the subsequent changes could have been anisotropic as well (fracturing, for example). In a multi-layer medium, the horizontal anisotropy may be of different directions in each layer. Such a viewpoint is wholly conventional in electromagnetic well-logging, where a medium and its bedding are thoroughly examined.

In earthbase electromagnetic soundings, the parametrization of a medium is much more scaled. We consider homogeneous multimeter layers (10–1000 m), which are the results of some degree of averaging, and in which, presumably, the horizontal anisotropy is offset (though it may also turn out to be significant). Some facts from applied electric prospecting bear evidence of the existence of such an anisotropy. The simplest and most well-known fact is the dependence of a signal on the orientation of units with grounding lines (provided, of course, that other factors are precluded).

Thus we now have a considerably more complicated geoelectric model of a medium (although it is still one-dimensional). Each layer is described by five parameters: thickness, three values of resistance (along directions *X*, *Y*, and *Z*), and the direction of the angle of horizontal resistances. Consequently, the mathematical apparatus becomes more complicated, and so the analytical approach almost reaches its limits.

The problem of interaction of a harmonic electromagnetic field with a biaxially-anisotropic medium has been examined earlier; see, e.g., [Tabarovskii and Epov, 1977; Sheen, 2005]. The study of solutions for a layered biaxially-anisotropy from the viewpoint of numerical implementation has required us to construct our own solution.

2. HOMOGENEOUS SPACE

Let us consider a harmonic electromagnetic field in a homogeneous anisotropic medium with biaxial anisotropy. The horizontal directions of the anisotropy do not agree with the directions of the axes of the system, and the spread in the angle α (Fig.1). In other words,

$$\hat{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}, \quad (1)$$

where

$$\sigma_{xx} = \sigma_1 \cos^2\alpha + \sigma_2 \sin^2\alpha, \quad \sigma_{yy} = \sigma_1 \sin^2\alpha + \sigma_2 \cos^2\alpha,$$

$$\sigma_{xy} = (\sigma_1 - \sigma_2)\cos\alpha\sin\alpha, \quad \sigma_{yx} = \sigma_{xy}, \quad \sigma_{zz} = \sigma_z.$$

Apart from the points with extraneous current, we consider a homogeneous system of Maxwell's equations (in a quasi-steady-state approximation):

$$\begin{cases} \mathrm{rot}\mathbf{H} = \hat{\sigma}\mathbf{E}, \\ \mathrm{rot}\mathbf{E} = \mathbf{i}\omega\mu\mathbf{H}, \\ \mathrm{div}\mathbf{H} = 0, \\ \mathrm{div}(\hat{\sigma}\mathbf{E}) = 0. \end{cases} \quad (2)$$

On applying the double Fourier transform to the problem in question, we will dispose of the lateral coordinates:

$$F(x, y, z) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\xi, \eta, z)e^{\mathrm{i}\eta x}e^{\mathrm{i}\eta y}d\xi d\eta. \quad (3)$$

Thus gives a homogenous problem in the space of harmonics:

$$\begin{cases} \mathrm{i}\eta h_z - (h_y)'_z = \sigma_{xx}e_x + \sigma_{xy}e_y, \\ (h_x)'_z - \mathrm{i}\xi h_z = \sigma_{yx}e_x + \sigma_{yy}e_y, \\ \mathrm{i}\xi h_y - \mathrm{i}\eta h_x = \sigma_z e_z, \\ \mathrm{i}\eta e_z - (e_y)'_z = \mathrm{i}\omega\mu h_x, \\ (e_x)'_z - \mathrm{i}\xi e_z = \mathrm{i}\omega\mu h_y, \\ \mathrm{i}\xi e_y - \mathrm{i}\eta e_x = \mathrm{i}\omega\mu h_z, \\ \mathrm{i}\xi\sigma_{xx}e_x + \mathrm{i}\xi\sigma_{xy}e_y + \mathrm{i}\eta\sigma_{yx}e_x + \mathrm{i}\eta\sigma_{yy}e_y = -\sigma_z(e_z)'_z, \\ \mathrm{i}\xi h_x + \mathrm{i}\eta h_y = -(h_z)'_z. \end{cases} \quad (4)$$

We first solve system (4) in the homogeneous domain, which does not contain sources (homogeneous problem). We single out from system (4) the four equations:

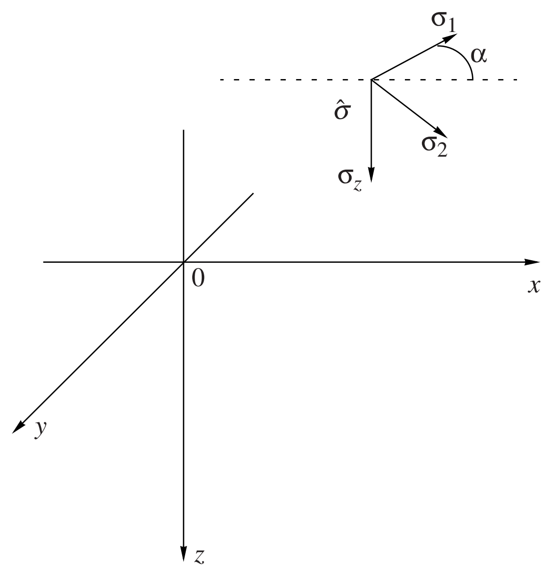

Fig. 1. Coordinate system and anisotropy model.

$$\begin{cases} \mathrm{i}\xi h_y - \mathrm{i}\eta h_x = \sigma_z e_z, \\ \mathrm{i}\xi\sigma_{xx}e_x + \mathrm{i}\xi\sigma_{xy}e_y + \mathrm{i}\eta\sigma_{yx}e_x + \mathrm{i}\eta\sigma_{yy}e_y = -\sigma_z(e_z)'_z, \\ \mathrm{i}\xi e_y - \mathrm{i}\eta e_x = \mathrm{i}\omega\mu h_z, \\ \mathrm{i}\xi h_x + \mathrm{i}\eta h_y = -(h_z)'_z. \end{cases} \quad (5)$$

They provide expressions of the horizontal components in terms of the vertical ones. In other words,

$$\begin{cases} \xi b_y - \eta b_x = j, \\ \xi k_1 e_x + \eta k_2 e_y = -j', \\ \xi e_y - \eta e_x = b, \\ \xi b_x + \eta b_y = -b', \end{cases} \quad (6)$$

where

$$k_1 = k_{xx} + \frac{\eta}{\xi}k_{yx}, \quad k_2 = \frac{\xi}{\eta}k_{xy} + k_{yy},$$

$j = k_{zz}e_z$, $b_\alpha = \mathbf{i}\omega\mu h_\alpha$, $k_{ab} = \mathbf{i}\omega\mu\sigma_{ab}$, $b = b_z$, f' denotes the derivative with respect to z; and we also write ξ and η for $\mathbf{i}\xi$ and $\mathbf{i}\eta$, respectively.

We have the following expressions for the horizontal components:

$$b_x = -\frac{1}{\lambda^2}(\eta j + \xi b'), \quad e_x = -\frac{1}{t^2}(\xi j' + \eta k_2 b),$$

$$b_y = \frac{1}{\lambda^2}(\xi j - \eta b'), \quad e_y = -\frac{1}{t^2}(\eta j' - \xi k_1 b), \quad (7)$$

here, $\lambda^2 = \xi^2 + \eta^2$ and $t^2 = \xi^2 k_1 + \eta^2 k_2$.

Accordingly, we first need to find b and j. To do so, we take from (4) two unused equations. For example (in our notation),

$$\begin{cases} \eta b - b'_y = k_{xx}e_x + k_{xy}e_y, \\ e'_x - \xi \dfrac{j}{k_{zz}} = b_y. \end{cases} \tag{8}$$

Substituting the expressions (7) for the horizontal components and observing that $C\exp(pz)$ is a particular solution for each component, we arrive at the following homogeneous system to determine b and j:

$$b\eta\left[\lambda^2 + \frac{\lambda^2}{t^2}(k_{xx}k_{yy} - k_{xy}k_{yx}) + p^2\right] + jp\xi\left(\frac{\lambda^2}{t^2}k_1 - 1\right) = 0, \tag{9}$$
$$bp\eta\left(\frac{\lambda^2}{t^2}k_2 - 1\right) + j\xi\left(\frac{\lambda^2}{t^2}p^2 + 1 + \frac{\lambda^2}{k_z}\right) = 0.$$

As the determinant of this system should be 0, we obtain the equation for p:

$$\begin{aligned} p^4 + bp^2 + c &= 0, \\ b &= \frac{t^2}{k_z} + \lambda^2 + \lambda^2\frac{\delta - k_1k_2}{t^2} + k_1 + k_2, \\ c &= \left(\frac{t^2}{k_z} + \frac{\delta}{k_z}\right)(\lambda^2 + k_z) = 0, \\ \delta &= k_{xx}k_{yy} - k_{xy}k_{yx}. \end{aligned} \tag{10}$$

We also obtain relations for b and j:

$$j = -\xi\eta p\frac{k_z(k_2 - k_1)}{\lambda^2 p^2 k_z + t^2(\lambda^2 + k_z)}b. \tag{11}$$

Equation (10) is a quadratic equation for p^2, and so we have four solutions: $\pm p_1$ and $\pm p_2$, which gives us the following general solution for each component:

$$f = C_1\exp(p_1z) + C_2\exp(-p_1z) + C_3\exp(p_2z) + C_4\exp(-p_2z).$$

3. BOUNDARY CONDITIONS

Let us now consider the horizontal boundary separating a medium into homogeneous regions of different conductivity tensors. At this boundary (which is free from extraneous currents), the horizontal components are continuous. In view of (4) and (7), this gives us the following adjointness conditions for the functions b and j and for their derivatives:

$$\begin{aligned} [b]\big|_{z = z_i} &= 0, \\ [b']\big|_{z = z_i} &= 0, \\ [j]\big|_{z = z_i} &= 0, \\ [j'/t^2]\big|_{z = z_i} &= \xi\eta\frac{k_{12}k_{21} - k_{11}k_{22}}{t_1^2t_2^2}b, \end{aligned} \tag{12}$$

here, k_α and t^2 are given above, and the subscripts 1 and 2 refer to the lower and upper medium, respectively, with respect to the boundary ($z = z_i$).

4. ACCOUNTING FOR THE SOURCE

Let us now take into account a source, which may be located also at the boundary separating two media at the point $z = z_0$. Our source is a magnetic dipole. A vertical dipole may be looked upon as a particular case of an arbitrary distribution of extraneous current in the plane $z = z_0$ (in amperes per meter). The adjointness conditions for the electromagnetic field at this boundary are physically apparent:

$$\begin{aligned} [H_x]\big|_{z = z_0} &= -j_y^E(x, y), \\ [H_y]\big|_{z = z_0} &= j_x^E(x, y), \\ [E_x]\big|_{z = z_0} &= 0, \\ [E_y]\big|_{z = z_0} &= 0. \end{aligned} \tag{13}$$

If we view the vertical magnetic dipole (VMD) as a small current loop, which, in polar coordinates, involves only $j_\varphi^{cm}(r)$, where $j_\varphi^{cm}(r) = I\delta(r - a)$, then we easily obtain, in the space of Fourier transforms, the following condition:

$$[h'_z]\big|_{z = z_l} = -M_z\lambda^2, \tag{14}$$

here, $M_z = \pi a^2$.

We next consider the horizontal magnetic dipole (HMD) as a particular case of an arbitrary distribution of surface extraneous magnetic current in the plane $z = z_0$ (in amperes). The adjointness conditions for the electromagnetic field at such a boundary are as follows:

$$\begin{aligned} [E_x]\big|_{z = z_0} &= \mathbf{i}\omega\mu j_y^M(x, y), \\ [E_y]\big|_{z = z_0} &= -\mathbf{i}\omega\mu j_x^M(x, y), \\ [H_x]\big|_{z = z_0} &= 0, \\ [H_y]\big|_{z = z_0} &= 0 \end{aligned} \tag{15}$$

or, in the space of harmonics,

$$[e_x]|_{z=z_0} = \mathbf{i}\omega\mu M_y,$$
$$[e_y]|_{z=z_0} = -\mathbf{i}\omega\mu M_x. \quad (16)$$

Based on these conditions and exploiting the expressions (7) of the horizontal components in terms of the vertical ones, gives us the adjointness conditions for the functions b and j in question j ($j = k_z e_z$, $b = \mathbf{i}\omega\mu h_я$):

(1) For the vertical magnetic dipole (M_z):

$$[b]|_{z=z_0} = 0,$$
$$[b']|_{z=z_0} = \mathbf{i}\omega\mu\lambda^2 M_z,$$
$$[j]|_{z=z_0} = 0, \quad (17)$$
$$[j'/t^2]|_{z=z_0} = \xi\eta\frac{k_{12}k_{21} - k_{11}k_{22}}{t_1^2 t_2^2}b;$$

(2) For the horizontal magnetic dipole (M_x):

$$\left[\frac{1}{t^2}(\eta j' - \xi k_1 b)\right]\Bigg|_{z=z_0} = \mathbf{i}\omega\mu M_x,$$
$$[b']|_{z=z_0} = 0,$$
$$[j]|_{z=z_0} = 0, \quad (18)$$
$$\left[\frac{1}{t^2}(\xi j' + \eta k_2 b)\right]\Bigg|_{z=z_0} = 0.$$

(3) For the horizontal magnetic dipole (M_y):

$$\left[\frac{1}{t^2}(\eta j' - \xi k_1 b)\right]\Bigg|_{z=z_0} = 0,$$
$$[b']|_{z=z_0} = 0,$$
$$[j]|_{z=z_0} = 0, \quad (19)$$
$$\left[\frac{1}{t^2}(\xi j' + \eta k_2 b)\right]\Bigg|_{z=z_0} = -\mathbf{i}\omega\mu M_y.$$

We recall that

$$k_1 = k_{xx} + \frac{\eta}{\xi}k_{yx}, \quad k_2 = \frac{\xi}{\eta}k_{xy} + k_{yy}$$

$k_{xy} = k_{yx}$, $k_{zz} \equiv k_z$, $j = k_{zz}e_z$, $b_\alpha = \mathbf{i}\omega\mu h_\alpha$, $k_{ab} = \mathbf{i}\omega\mu\sigma_{ab}$, $b = b_z$, and f' denotes the derivative with respect to z; and we also write ξ and η for $\mathbf{i}\xi$ and $\mathbf{i}\eta$, respectively.

Formulas (17)–(19) express the adjointness conditions in the most general manner, when a source is positioned at the real boundary of separation with various parameters. If there is no source, then we arrive at the above conditions (12) for crossing the boundary without a source. On the other hand, if the source is in a homogeneous medium, then the conditions can be written as:

$$[b]|_{z=z_0} = \mathbf{i}\omega\mu(\xi M_x + \eta M_y),$$
$$[b']|_{z=z_0} = \mathbf{i}\omega\mu\lambda^2 M_z,$$
$$[j]|_{z=z_0} = 0, \quad (20)$$
$$[j']|_{z=z_0} = -\mathbf{i}\omega\mu(\eta k_2 M_x - \xi k_1 M_y).$$

5. RECURSION

We consider one of the N ($i = 1, 2, 3, \ldots, N$) layers of the horizontally-layered geoelectric model. In each homogeneous layer (we drop the index i) and we solve the quadratic equation in p_2:

$$p^4 + bp^2 + c = 0,$$
$$b = \frac{t^2}{k_z} + \lambda^2 + \lambda^2\frac{\delta - k_1 k_2}{t^2} + k_1 + k_2,$$
$$c = \left(\frac{t^2}{k_z} + \frac{\delta}{k_z}\right)(\lambda^2 + k_z) = 0,$$
$$\delta = k_{xx}k_{yy} - k_{xy}k_{yx}.$$

There are four solutions: $\pm p_1$ and $\pm p_2$. This suggests that a solution in a homogeneous layer should be looked for in the form

$$b = C_1\exp(p_1 z) + C_2\exp(-p_1 z) + C_3\exp(p_2 z) + C_4\exp(-p_2 z)$$
$$b' = C_1 p_1\exp(p_1 z) - C_2 p_1\exp(-p_1 z) + C_3 p_2\exp(p_2 z) - C_4 p_2\exp(-p_2 z),$$
$$j = C_1\alpha_1\exp(p_1 z) - C_2\alpha_1\exp(-p_1 z) + C_3\alpha_2\exp(p_2 z) - C_4\alpha_2\exp(-p_2 z),$$
$$j' = C_1 p_1\alpha_1\exp(p_1 z) + C_2 p_1\alpha_1\exp(-p_1 z) + C_3 p_2\alpha_2\exp(p_2 z) + C_4 p_2\alpha_2\exp(-p_2 z),$$

where we utilized the bond

$$j = \alpha b,$$
$$\alpha = -\xi\eta p\frac{k_z(k_2 - k_1)}{\lambda^2 p^2 k_z + t^2(\lambda^2 + k_z)}$$

which occurs in a biaxially-anisotropic medium. The coefficients C_1, C_2, C_3, and C_4 can be expressed in terms of the values of our functions b, b', j and j' on the lower (or upper) boundary of the layer and so obtain the algorithm for the recalculation of the function from one layer to another with downward (or upward) motion along the domain. Thus,

$$b_- = pc_1 - rs_1 + sc_2 - ts_2,$$
$$b'_- = pp_1c_1 - rp_1s_1 + sp_2c_2 - tp_2s_2,$$
$$j_- = p\alpha_1c_1 - r\alpha_1s_1 + s\alpha_2c_2 - t\alpha_2s_2, \quad (21)$$
$$j'_- = p\alpha_1p_1c_1 - r\alpha_1p_1s_1 + s\alpha_2p_2c_2 - t\alpha_2p_2s_2,$$

Table 1

Layer number	ρ_x, Ω m	ρ_y, Ω m	ρ_z, Ω m	Angle, degree	Layer thickness, m
0	1	2	4	30	Upper half-space
1	2	3	20	0	3
2	0.1	0.2	0.3	–30	Lower half-space

where $s_{1,2} = u\sinh(p_{1,2}h)$, $c_{1,2} = \cosh(p_{1,2}h)$, $u = 1$, if the motion is downward (i.e., we express the values on the lower level in terms of the values on the upper level), and $u = -1$ for the upward motion; h is the recalculation step along z-axis (in particular, the layer thickness), and

$$p = \frac{bp_2\alpha_2 - j'}{p_2\alpha_2 - p_1\alpha_1}, \quad r = \frac{b'\alpha_2 - jp_2}{p_2\alpha_1 - p_1\alpha_2},$$

$$s = \frac{b'\alpha_1 - jp_1}{p_2\alpha_1 - p_1\alpha_2}, \quad t = \frac{bp_1\alpha_1 - j'}{p_2\alpha_2 - p_1\alpha_1}.$$

We now can "move" upwards and downwards along the domain using the recalculation (21) relative to step h and crossing the boundaries under conditions (12). The algorithm then works by defining the solution in the upper half-spaces as $b = A\exp(p_{01}z) + B\exp(p_{02}z)$, and, in the lower half-space, as $b = C\exp(-p_{N1}z) + D\exp(-p_{N2}z)$ and so we move to the source both from above and from below. The coefficients A, B, C, and D are found by satisfying the conditions (17)–(19) or (20) at the source.

6. CALCULATIONS IN THE FREQUENCY DOMAIN AND TESTING

The BIAMAG2 (FORTRAN) program was written based on the above algorithm. The key problems of the new algorithm as compared with the algorithms involv-

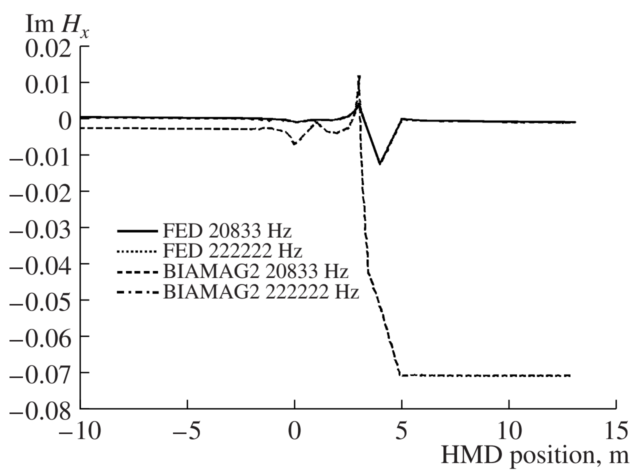

Fig. 2. Calculations by the BIAMAG2 program versus calculations by the FED method.

ing the conventional anisotropy are bound up with the following two facts. The first is that, instead of the one-dimensional Hankel transform, we now must invoke the double Fourier transform. This drastically increases the computing time. The second difficulty arises due to the relation between the electrical mode and the magnetic mode (or more precisely, because of the relation between h_z and e_z). Consequently, the numerical implementation of recursive algorithms in multi-layer media becomes less stable and calls for some supplementary measures.

At this stage, the constant comparison of the results calculated by the above program with the results of calculations based on the finite element methods using the FEMCYL3D program was of great value. The FEMCYL3D fractured medium (developed by Baker Hughes) is designed for three-dimensional mathematical modeling of electromagnetic well-logging tools [Bespalov, 2007]. This program performs the numerical computation of time-harmonic Maxwell's equations. The discretization of the problem was made by the finite element method on a cylindrical grid, uniform in azimuth, with the employment of the "coastal" basic Nedelec functions adapted to the cylindrical geometry. The resulting algebraic system is settled by means of the (generalized) iterative method of minimal residuals with the employment of the preconditioner used for the improvement of convergence. As a preconditioner, we use the difference operator which is as close as possible to the operator of the problem, and which, however, does not exhibit azimuthal dependence of the elements. This fact enables us, in order to invert the preconditioner, to invoke the discrete fast Fourier transform with respect to the azimuth, which turns the three-dimensional grid problem into a collection of mutually independent two-dimensional problems. These two-dimensional problems are settled by the direct Gauss method involving optimal ordering by the method of nested partitions.

The FEMCYL3D program by no means confines neither the character nor the spatial distribution of electromagnetic coefficients of a medium: at each point the conductivity, as well as the dielectric and diamagnetic permittivity, may be arbitrary total tensors.

Consequently, the debugging and testing of the BIAMAG2 program were developed from comparing the calculation results with the results obtained by the finite element method (the finite element discretization, FED) calculated for a three-layer medium with different directions of anisotropy in each layer (Table 1).

Figures 2 and 3 illustrate this comparison for two frequencies for the imaginary parts of the magnetic components: for H_x relative to the horizontal (M_x) magnetic dipole and for H_z relative to the vertical magnetic dipole. The horizontal axis shows positions of the source, which is located one meter below the point of observation in the vertical direction. The first boundary of the medium is located at level 0.

The test findings show that the algorithm developed is quite capable (in the applied sense) of computing all the components of a harmonic electromagnetic field of an inclined magnetic dipole in a biaxially-anisotropic medium (even if we ascribe all the differences to the BIAMAG2 program). Also, the calculations (BIAMAG2 and FED) were made precisely in the class of "biaxial" models, and the discrepancy between them is very much smaller than the influence of the horizontal anisotropy itself.

Another test comparison of some interest from the standpoint of applicability of the algorithm developed under conditions of ground geoelectrics was carried out for the model shown in Fig. 4.

In this case, the set-up was stationary, and the calculations were carried out for a series of frequencies. A source (VMD) was placed at the boundary between an almost isolating half-space and a conductive biaxially-anisotropic layer underlaid by a conductive isotropic half-space. The field H_z was observed at the second boundary. As in the previous example, the results of the calculations were compared with the results obtained by the finite element method.

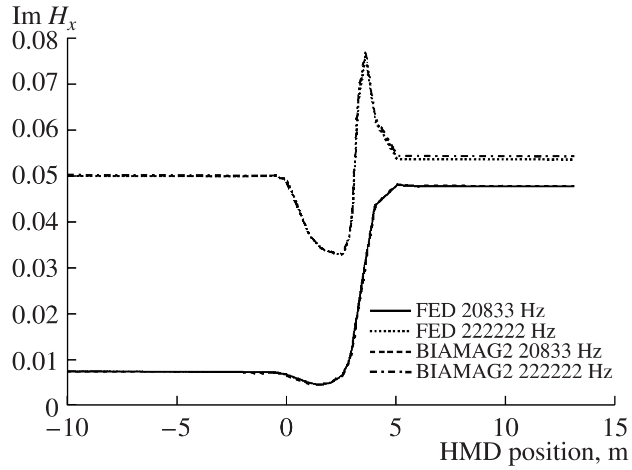

Fig. 3. Calculations by the BIAMAG2 program versus calculations by the FED method.

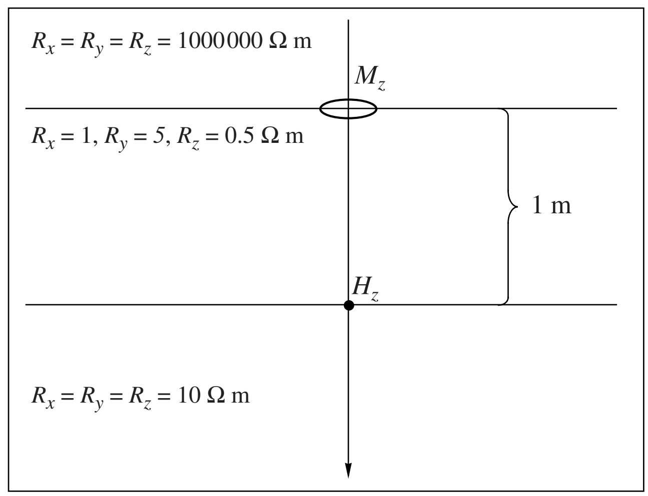

Fig. 4. Medium model, source location, and observation points in the test calculation.

7. CALCULATIONS IN THE TIME-DOMAIN

The BIAMAG2 program was written mostly for purposes of induction well-logging. Based on this program, we can easily create a procedure for the calculation of transient fields. However, in the time-domain, we did not have the possibility to conduct a nontrivial test (i.e., precisely for a biaxially-anisotropic model). Nevertheless, we are pretty confident in the results obtained, since our treatment is based on a well-tested procedure in the frequency domain and on perfectly standard procedures of the numerical Fourier transform, which we have been using for a long time in the well-known system for processing electric prospecting data ("Podbor" and "Vybor-3S"). We shall illustrate a new phenomenon that occurs in a biaxially-anisotropic medium by means of this particular transient mode.

Table 2 illustrates a three-layer cut whose second layer exhibits biaxial anisotropy. As a source we consider a vertical magnetic dipole (M_z= 1013 A m^2) on the daylight surface. Observation takes place at a depth of 200 meters; i.e., on the second boundary at the point with coordinates x =200 m and y = 200 m (inasmuch as the vertical resistances are the same below and above the boundary). The response waveband ranges from 1 ms to 5 s. Figure 5 shows the transient curves for all three components E_x, E_y and E_z of the electrical field. We notice that in this case the vertical component (which is absent in the conventional isotropic case or in a medium with one anisotropic axis) is fully comparable with the horizontal ones.

Even geophysicists, not just electric prospectors, are well aware of such facts of electric prospecting as the excitement of only the horizontal system of secondary

Table 2

Layer number	Layer thickness, m	ρ_x, Ω m	ρ_y, Ω m	ρ_z, Ω m
1	200	5	5	5
2	100	0.1	1	5
3	∞	100	100	100

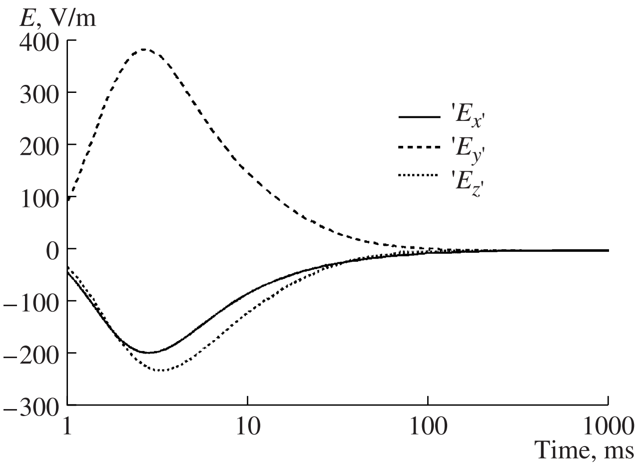

Fig. 5. Transient curves of all components of the gradient.

earth currents in the case when a source (a current loop) is located on a horizontal surface (daylight surface, for example). However, as we see, this is no longer true for a biaxially-anisotropic medium. Even though this result can be easily obtained theoretically, our calculations give a numerical estimate showing that these influences may be of considerable importance.

8. CONCLUSIONS

The presented mathematical software for geoelectric problems in media with biaxial conductivity, effected by transient electromagnetic sounding and based on an analytical solution, is quite compact (in spite of the difficulties in its numerical implementation), does not involve large computational resource, is off-line, and is accessible by a wide range of electric prospectors. One of the features of biaxial anisotropy alluded to above—the appearance of a vertical component of the electrical field (quite a substantial one) on excitation by the horizontal current loop—may have various and unexpected consequences, which are useful to bear in mind in the qualitative interpretation of data by means of (for example) transient electromagnetic sounding. However, this question depends essentially on the presence and dispersion of the biaxial anisotropy of conductivity in a real geological environment.

In this paper, we have considered induction excitement. The horizontal anisotropy effect on the field of the ground power line is of even greater practical importance. We are going to outline the results for grounded sources in a subsequent paper in the very near future.

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