Formation of the macroanisotropic geoelectric parameters of a thin-layered geologic medium and the resolution of electrical prospecting

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Abstract

The accuracy (detail, resolution) of controlled-source electrical prospecting is a combination of several disparate elements. They include the model framework used in the interpretation. A real geologic medium has a complex structure. Sedimentary rocks have a layered structure with fractal properties: The layers split into smaller ones. Large-scale geoelectrical studies (for example, electrical prospecting) require a proper geoelectrical model. By necessity, the 1D geoelectrical model in electrical prospecting is horizontally layered, with thick (hundreds of meters) homogeneous layers, whereas a fine structure is neglected. To study some aspects of this problem, we performed a set of numerical experiments. They were aimed at studying the TEM response from a formation consisting of many thin layers with random geoelectric parameters, mainly resistivity.

Keywords: TDEM method; macroanisotropy; interpretation; geoelectrical model; random parameters; mathematical modeling

Introduction

The accuracy (detail, resolution) of controlled-source electrical prospecting is a combination of several disparate elements, to name only three: the preset parameters and, consequently, type of the field used (physical basis); the technical practice (response recording); and the model framework used in the interpretation. Let us consider only the last aspect. It is evident from direct observations that a real geologic medium has a complex structure. Sedimentary rocks have a layered structure with fractal properties: The layers split into smaller ones. This is illustrated well by Fig. 1, but, in fact, such a pattern is observed everywhere.

Certainly, a layered structure presupposes different physical, including EM, parameters of each layer. The results of the small-scale geoelectrical studies of the geologic medium (electrical logging) are shown in Fig. 2. This is the usual shape of resistivity logs. As we can see, resistivity varies in quite an intricate way. Note that even these slight variations result from the averaging of the parameters of thinner layers.

Large-scale geoelectrical studies (for example, electrical prospecting) require a proper geoelectrical model. It is clear that we cannot model the structure of a real geologic medium in great detail, for at least two reasons. First, our large-scale physical experiment, by necessity, has the spatiotemporal characteristics (frequency) which smooth over small details. Second (this reason is closely related to the first one), it is impossible to do the interpretation with a large number of parameters.

Thus, by necessity, the 1D geoelectrical model in electrical prospecting is horizontally layered, with thick (hundreds of meters) homogeneous layers (rarely >10). Such models, with ~20 parameters (ten resistivity values, ten power values), are used for interpreting electrical prospecting data. For example, the thin-layered medium shown on the logging trace (Fig. 2) will probably be interpreted in electrical prospecting as a three-layered one, with boundaries at depths of 200 and 400 m.

Note that the fine structure of the layer (including thin layers, which show wide variation not only in resistivity but also in dielectric and magnetic permeability) is, as a rule, neglected. In this case, the discrepancy between the model and the real medium might be manifested in the frequency–time variance of “true” resistivity. Can we speak of geometric, or
structural, or model variance? However, having attributed this variance to induced polarization (IP) and determined the values of the apparent IP parameters, we might come to absolutely wrong conclusions about the composition of the medium.

To study some aspects of this problem, we performed a set of numerical experiments. They were aimed at studying the TEM response from a formation consisting of many thin layers with random geoelectric parameters, mainly resistivity. The values of the fields obtained were compared with the field values of an equivalent homogeneous layer. What “equivalent” means, will be explained below.

Note that, in connection with this, the excitement of current in the medium by inductive and galvanic sources (or, actually, the geoelectromagnetic processes of electric and magnetic settling) shows considerable differences. Therefore, we study equivalent thin-layered models under the effect of three field types: magnetic, electric, and mixed.

**Synthesizing a thin-layered random model and an equivalent homogeneous layer**

It is not a simple question whether thin-layered and homogeneous formations are equivalent. First, note that these are different physical objects, one of them being much more complex than the other. Equivalence can clearly be only relative and arbitrary not absolute. We should keep in mind that equivalence is considered here in terms of the interaction between the EM field and the medium. That is, we should consider the field reactions to thin-layered and homogeneous objects and compare their EM parameters at which the reactions are the same. Furthermore, it is clear that this equivalence depends on the EM-field configuration and frequency. For example, the absence of the electric-field component normal to the boundaries of thin conducting layers produces only tangential current. In this case we are interested only in the longitudinal conductivity of the medium. All multilayered formations (including the homogeneous one) with the same longitudinal conductivity will be equivalent to different extent, different as transverse resistivity may be. Certainly, this situation will also depend on the EM-field frequency.

The problem arises in different applications of the EM field along with geoelectrics. For a low-frequency region, which is used in surface geoelectrics, in quasi-stationary approximation, the results of (Rytov, 1955) are fundamental to a nonmagnetic medium.

So, for the longitudinal and transverse resistivity of an equivalent homogeneous layer of thickness $H$, we have

$$\rho_L = \frac{H}{S}, \quad \rho_T = \frac{T}{H}, \quad (1)$$

where $\rho_L$ is longitudinal (tangential, horizontal) resistivity; $\rho_T$, the transverse (normal, vertical) resistivity of an equivalent homogeneous anisotropic layer; and

$$S = \sum_{i=1}^{N} h_i / \rho_L, \quad T = \sum_{i=1}^{N} h_i \rho_L; \quad (2)$$

Here, $S$ is the total longitudinal conductivity; $T$, the total transverse resistivity of a formation consisting of $N$ thin isotropic layers of thickness $h_i$ and resistivity $\rho_i$ ($i = 1, 2, 3, ..., N$).

Surely, geoelectricians have long known about the equivalence of multilayered formations with the same $S$ and $T$ without Rytov’s theory and understood the approximate, asymptotic, character of formulae (1) and (2) from practice and forward modeling. Now, on the basis of numerical modeling, we can estimate the difference resulting from the replacement of a real random thin-layered formation by a homogeneous one.

We used the algorithm for synthesizing a thin-layered rock unit of thickness $H$ and with total longitudinal conductivity $S$ and total transverse resistivity $T$. The problem was not complex, excluding the infinite set of possible solutions, which we had to choose from in a certain way. We limited the thickness of the thin layers to only three values. Resistivity was determined by a random (pseudorandom) number generator (a procedure from standard software libraries).

Consider the example of a 100-m-thick formation consisting of 1000 thin layers, with a longitudinal resistivity value...
Fig. 2. Resistivity curve reflecting the thin-layered structure of the geologic medium.

Fig. 3. An example of a random medium as a longitudinal-conductivity plot.
of 2 Ω·m and a transverse resistivity value of 5 Ω·m. Longitudinal conductivity is shown in Fig. 3, and the resistivity plot is unrepresentative because of individual random large values. The conductivity of individual layers is limited, because we have taken the minimum resistivity value to be 0.05 Ω·m. Correspondingly, the maximum conductivity is limited to 20 S/m (Fig. 3).

**Numerical experiments**

The idea of the numerical experiments is quite simple. We place transmitters of different types operating in the pulse mode on the formation, which was formed randomly from many (1000) thin layers of different conductivity, so that Rytov’s values of longitudinal and transverse resistivity are equal to the preset ones. The response is recorded on the lower surface (Fig. 4), unlike the actual placement on the day surface. This is done for two important reasons. First, we would like all the thin layers to be under equal conditions. In the actual placement (transmitter and receiver on the day surface), the upper layers are much more important for the response settling than the lower ones. Second, the vertical separation of the transmitter and receiver facilitates the algorithmic problem related to thin layers and permits doing a fast calculation, which is very important for a multiple search.

So, we record the response and compare it with the stored response of a homogeneous layer with the same total power, longitudinal conductivity, and transverse resistivity values. If the deviation is larger than that recorded previously, it becomes the next index and the cycle “new medium–calculation–comparison” goes on indefinitely, detecting media which yield a response with an increasing deviation from the homogeneous medium. Having detected an interesting medium, we can test it using usual electrical prospecting equipment, as will be demonstrated below.

Furthermore, we should decide what transmitters and receivers to use. Limiting ourselves to one transmitter–receiver means limiting the value of our experiment. Neither is it possible to try all the combinations. We have a fundamental approach requiring that three transmitters–receivers be tested which will record signals related to the TE-polarization of the EM field, TM, and the most widely used combination TE + TM. Thus, here we are using the following transmitters–receivers.

1. The transmitter is a horizontal electrical line (HEL, 10 m long, current 1 A), and component \( dB_z/dt \) is measured at the dipole equator (spacing 100 m). Although the HEL excites fields of both types of polarization (TE, TM), this component belongs only to the TE-field (or magnetic-type field);
2. The transmitter is a circular electrical dipole (CED, radius 10 m, current 1 A), and the component measured is \( E_x \) (spacing 100 m). This is certainly pure TM-polarization (or electric-type field);
3. The transmitter is a HEL (10 m long, current 1 A), but component \( E_z \) is measured on the dipole axis (spacing 100 m). This apparatus is conventional and popular; it records the signal belonging to two types of polarization (TM+TE).

**Mathematical methods for calculating response from multilayered media**

The mathematical methods for calculating the TEM field are well-known, especially the frequency-domain solution followed by transformation (Dmitriev, 1968; Mogilatov, 2002; Van’yan, 1965). The Geoelectrics Laboratory of the Trofimuk Institute of Petroleum Geology and Geophysics has a long tradition of the computer application of this method to different field sources. We also developed the method offered by A.N. Tikhonov and O.A. Skugarevskaya (1950), also known as the time-domain solution, or the transient-harmonics method. If the layers are numerous (up to 1000), a new aspect appears. We had to validate and enhance recursive algorithms for the integrand (univariate) function as well as study their stability in the case of very long recursions.

According to analysis, magnetic-type recursions are quite stable, whereas electric-type ones have problems, because interrupted boundaries are crossed, as is not the case with magnetic-type recursions. Nevertheless, a modification permitted quite reliable calculations of long recursions of both types. The convergence of the Hankel integrals was, as we said, facilitated greatly by the localization of the transmitter and receiver on different sides of the formation.

The validation consisted in recursion up and down the medium (the values of the function and its derivative had to be constant) and a calculation for a pseudolayered medium with a larger number (~1000) of fictitious boundaries. Also, the results were compared with the calculations by a whole different algorithm, based on Tikhonov’s solution.

**Results**

The results are tentative as yet. We launched all three programs for each transmitter-receiver, for several days each. The experiment was interrupted for analysis and corresponding correction. Note that Rytov’s equivalence operated very well: As early as a day later, we almost stopped obtaining responses.
whose deviations from that of the homogeneous medium (equivalent formation, according to Rytov) had to be recorded as maximum.

Our results amounted themselves to the following. For HEL–$dB/dt$ (pure TE-field), the mean square deviation from the homogeneous formation ($H = 100$ m; $S = 50$ S) was 13%. A thin-layered medium with the same total power and conductivity is shown in Fig. 5, and it is responsible for this deviation. The plot shows resistivity on a logarithmic scale.

Nothing interesting is seen. We will analyze this medium below. A TDEM curve is compared with a homogeneous-layer curve in Fig. 6, $a$, with a 13% deviation.

For HEL–$E_x$ (TE + TM), 21% was obtained (Fig. 6, $b$).

Finally, 33% was obtained for CED–$E_r$ (TM) (Fig. 6, $c$).

Now the question arises how to interpret these results. We deem it premature to assign the values obtained to the accuracy of the models and of the TDEM method in general. Let us trace the distribution of longitudinal conductivity within the thin-layered formations. In Fig. 7, plots of the depth increment of the total longitudinal conductivity are shown for all three formations responsible for an anomalous response from three types of transmitters–receivers.

Straight lines show the plot for a homogeneous formation. It is seen that conductivity was just randomly redistributed in the case of TE (Fig. 7, $a$): It is excessive in the lower part of the layer and insufficient in its upper part. For TE + TM and TM (Fig. 7, $b$, $c$), the situation is not so obvious.

Let us analyze the situation in terms of electrical prospecting. For the medium obtained in the TE-experiment, we calculate a synthetic TDEM curve for a usual near-field TDEM apparatus with coaxial loops (transmitter $80 \times 80$ m, receiver $1 \times 1$ m with an effective moment of 1000 m$^2$, current 1 A). To overcome the poor convergence of the Hankel integrals (now the transmitter and receiver are located on the day surface), we used the method of complex integration path deformation, neglecting the calculation time. The independent expert A.K. Zakharkin made a standard layer interpretation, which yielded the following model:

<table>
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<th>Layer no.</th>
<th>Resistivity, $\Omega \cdot m$</th>
<th>Thickness, m</th>
</tr>
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<tr>
<td>1</td>
<td>3.73</td>
<td>6.93</td>
</tr>
<tr>
<td>2</td>
<td>2.16</td>
<td>49.6</td>
</tr>
<tr>
<td>3</td>
<td>1.79</td>
<td>47.4</td>
</tr>
<tr>
<td>4</td>
<td>$\infty$</td>
<td>$\infty$</td>
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</table>

Apparent resistivity curves for a near-field TDEM apparatus are shown in Fig. 8, $a$. We see consistent field (1001 layers) and adjusted four-layered curves as well as a curve for an equivalent homogeneous layer ($\rho = 2 \ \Omega \cdot m; \ H = 100$ m).

Thus, the problem has seemingly been solved in a simple and satisfactory way. The near-field TDEM method “sees” this layer as consisting of three individual layers.

However, if we imagine this layer as part of a thicker (~1 km) layered medium, which is studied in common near-field TDEM practice, it will most likely be considered homogeneous.
Fig. 6. TDEM curves in the experiment: a, TE; b, TE + TM; c, TM. 1, homogeneous formation; 2, 1001-layered formation.

Fig. 7. Plots of the total longitudinal conductivity vs. depth for the formations obtained in the TE (a), TE + TM (b), and TM (c) experiments. See legend in Fig. 6.
The model looks even more arbitrary in the case of the medium obtained from the TM-experiment. The medium which yields up to 33% deviation from Rytov’s equivalent in the TM-field is almost indistinguishable from the homogeneous equivalent in the near-field TDEM method (Fig. 8, b).

Conclusions

We are prone to regard our results as tentative. We began this work long ago, impressed by frequent reports of high-resolution phenomena in the field. What is more, high-resolution electrical prospecting has actually been introduced as a method along with common electrical prospecting. However, high-resolution phenomena allow of different interpretations, suggesting that there is no right, if any, explanation as yet. Certainly, the diffusion of our field in structural electrical prospecting is a serious obstacle to detail. The field diffuses downward from the source and diffuses every (if any) fine deep abnormal effect upward. In this case what will remain of it on the day surface, on which we record the response? Nevertheless, we would like to make a contribution by studying the effect of model inconsistency in the hope that numerical experiments with very complex (1D) media will “capture” the effects (maybe, of resonance character) which have eluded theorists. So far we have understood only that models are arbitrary and electrical prospecting data are somewhat uncertain. However, our numerical experiment is of very modest scale as yet, and it is to be continued. Also, our present easily explicable results might be due to the neglect of the magnetic- and dielectric-permeability distribution in the thin layers (displacement current was ignored). These parameters should also be taken into account to be on the safe side.

References


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